**Signals & Systems**

**EEE-223**

Lab # 08



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**LAB # 08**

**Properties of Convolution**

**Lab 08-** **Properties of Convolution**

**Pre-Lab Tasks**

**8.1 Properties of Convolution:**

In this section, we introduce the main properties of convolution through illustrative examples.

* Commutative Property

For two signals and the commutative property stands; that is equation 8.1, given as



**Example:**

Verify the commutative property of the convolution supposing that and .

The left side of 8.1, i.e., the signal is computed and plotted first while the right side of 8.1, namely,  is computed and plotted.

|  |  |  |
| --- | --- | --- |
| Commands | Results | Comments |
| t=0:0.01:5;  h1=ones(size(t));  h2=2\*exp(-2\*t);  y=conv(h1,h2)\*0.01;  plot(0:0.01:10,y),grid on  title('h\_1(t)\*h\_2(t)') | 11.bmp | The left side of 8.1. |
| z=conv(h2,h1)\*0.01;  plot(0:0.01:10,y),grid on  title('h\_2(t)\*h\_1(t)') | 12.bmp | The right side of 8.1. |

* Associative Property

For three signals,  and  the associative property stands; that is 8.2, given as,



**Example:**

Verify the associative property of the convolution supposing that and; and.

For the left side of 8.2, which is, first the convolution is computed. Next, the signal is defined in the same time interval with (at) and the result of their convolution is plotted.

|  |  |  |
| --- | --- | --- |
| Commands | Results | Comments |
| t=0:0.01:5;  x=ones(size(t));  h1=1/pi\*t;  y1=conv(h1,x)\*0.01;  h2=2\*exp(-2\*t);  th2=5.01:0.01:10;  hh2=zeros(size(th2));  h2=[h2 hh2];  y=conv(h2,y1)\*0.01;  plot(0:0.01:20,y), grid on  title('h\_2(t)\*(h\_1(t)\*x(t)') | 13.bmp | The left side of 8.2. |

The right side of 8.2, which is, first the convolution is computed. Next the signal  is defined in the same time interval with (at) and the result of their convolution is plotted.

|  |  |  |
| --- | --- | --- |
| Commands | Results | Comments |
| t=0:0.01:5;  h1=1/pi\*t;  h2=2\*exp(-2\*t);  z1=conv(h1,h2)\*0.01;  x=ones(size(t));  tx=5.01:0.01:10;  xx=zeros(size(tx));  x=[x xx];  z=conv(z1,x)\*0.01;  plot(0:0.01:20,z), grid on  title('(h\_2(t)\*h\_1(t))\*x(t)') | 14.bmp | The right side of 8.2. |

* Distributive Property

For three signals,  and  the distributive property stands; that is 8.3, given as



**Example:**

Illustrate the distributive property of convolution by using the signals; ; and .

In the similar vein to two previous examples, the left side of 8.3, that is, is compared to the right side of 7.3, that is, .

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| --- | --- | --- |
| Commands | Results | Comments |
| t=0:0.01:5;  x=ones(size(t));  h1=cos(pi\*t);  h2=2\*exp(-2\*t);  h=h1+h2;  y=conv(h,x)\*0.01;  plot(0:0.01:10,y),grid on  title('[h\_1(t)+h\_2(t)]\*x(t)') | 15.bmp | The left side of 8.3. |
| t=0:0.01:5;  x=ones(size(t));  h1=cos(pi\*t);  h2=2\*exp(-2\*t);  z1=conv(h1,x)\*0.01;  z2=conv(h2,x)\*0.01;  z=z1+z2;  plot(0:0.01:10,z),grid on  title('h\_1(t)\*x(t)+h\_2(t)\*x(t)') | 16.bmp | The right side of 8.3. |

* Identity Property

Ifis the Dirac delta function, then for any signal  the following expression is true:



This property is straight forwardly proven from the definition of Dirac function. Nevertheless, an example will be provided in lab of discrete time convolution.

**8.2 Interconnections of Systems:**

Systems may be interconnections of other sub-systems. The basic interconnections are the cascade, the parallel, the mixed and the feedback. The block diagrams are illustrated in Figure 8.1.

Input





Output

System 

System 

(a)

Output

Input

System 





System 

(b)

Input

Output

System 





System 

System 

(c)

Output





Input

System 

System 

(d)

**Figure 8.1:** Interconnections of (sub) systems: (a) Cascade, (b) Parallel, (c) Mixed, and (d) Feedback

When two systems  and are cascade (or serially) connected (Figure 8.1a), the output of the first system is the input of the second system. The block diagram off two parallel interconnected systems is presented in Figure 8.1b. The same input signals are applied to the two parallel-connected systems and the output of  and are combined to generate the overall output. The mixed interconnection is a combination of cascade and parallel interconnections. In the block diagram of Figure 8.1c, systems  and  are parallel connected, and their output is input to the cascade-connected system. Finally in Figure 8.1d the feedback interconnection block diagram is depicted. The output of  is input to, while the output ofis fed back to  and combined with the input signal produce the overall output of the system.

The interconnected subsystems can be considered as one system, i.e., an equivalent system described by one overall impulse response. In order to compute the output and the impulse response of the equivalent overall system for the various types of interconnections, suppose that the subsystems is described by the impulse response  and the subsystem by the impulse response. Finally, let be the input signal.

* Cascade interconnection

The output of is input to . Thus, the output of the equivalent systems is computed as ; that is, the input signal is first convoluted with the impulse response of  and the computed output is convoluted with the impulse response of .

|  |  |  |
| --- | --- | --- |
| Commands | Results | Comments |
| t=0:0.01:3;  x=ones(size(t));  h1=t.\*exp(-3\*t);  y1=conv(x,h1)\*0.01;  t1=0:0.01:3;  h2a=t1.\*cos(2\*pi\*t1);  t2=3.01:0.01:6;  h2b=zeros(size(t2));  h2=[h2a h2b];  y=conv(y1,h2)\*0.01;  plot(0:0.01:12,y,'linewidth',2),grid on  legend('y(t)') |  | Graph of the output |

To compute the impulse response of the overall system, the associative property of the convolution is applied. More specifically, applying the associative property to the output relationship  yields. Consequently, the impulse response of the overall equivalent system, if the subsystem, are cascade connected, is given by  . Straightforwardly, the systems response of  is given by.

|  |  |  |
| --- | --- | --- |
| Commands | Results | Comments |
| t=0:0.01:3;  h1=t.\*exp(-3\*t);  h2=t.\*cos(2\*pi\*t);  h=conv(h1,h2)\*0.01;  plot(0:0.01:6,h,'linewidth',2), grid on;  legend('h(t)') |  | The impulse response  is obtained by |
| t1=0:0.01:3;  t2=3.01:0.01:6;  x1=ones(size(t1));  x2=zeros(size(t2));  x=[x1 x2];  y=conv(x,h)\*0.01;  plot(0:0.01:12,y,'linewidth',2),grid on;  legend('y(t)') |  | The output of the system  is computed from the convolution between the input signal and the overall impulse response, which was derived using the associative property. |

The two graphs are identical; hence, the computation of the impulse response and the output signal of the equivalent system are correct.

* Parallel Interconnection

In this type of interconnection, the same input is applied to both subsystems. The two outputs of subsystems are added to obtain the final output. The mathematical expression is.

|  |  |  |
| --- | --- | --- |
| Commands | Results | Comments |
| t=0:0.01:3;  h1=t.\*exp(-3\*t);  h2=t.\*cos(2\*pi\*t);  x=ones(size(t));  y1=conv(h1,x)\*0.01;  y2=conv(h2,x)\*0.01;  y=y1+y2;  plot(0:0.01:6,y,'linewidth',2), grid on;  legend('y(t)') |  | The response of the systems to the input signal is computed by adding the outcome of the convolutions between the input signal and the impulse response of the subsystems; that is, it is computed as |

To compute the impulse response of the overall system the distributive property of the convolution is applied. More specifically, applying the distributive property to the output relationship  yields. Consequently, the impulse response of the equivalent system when the subsystems are parallel connected is given by . Straightforwardly, the output of the system is given by. To verify the conclusion, we consider the same signals used in the cascade interconnection case, namely,, and .

|  |  |  |
| --- | --- | --- |
| Commands | Results | Comments |
| t=0:0.01:3;  h1=t.\*exp(-3\*t);  h2=t.\*cos(2\*pi\*t);  h=h1+h2;  plot(0:0.01:3,h,'linewidth',2), grid on;  legend('h(t)') |  | The overall impulse response of the system is the sum of the impulse responses of the subsystems; that it is computed by |
| x=ones(size(t));  y=conv(x,h)\*0.01;  plot(0:0.01:6,y,'linewidth',2),grid on;  legend('y(t)') |  | The output of the system is computed from the convolution between the input signal and the impulse response of, which was derived using the distributive property. |

The graphs of the system response are identical; hence, our computation of the impulse response and the output of the equivalent system is accurate.

Note: The implementation of the mixed interconnection is left as an exercise to the students.

**8.3 Stability Criterion for Continuous Time Systems:**

The concept of stability was introduced in lab session 5. A system is bounded-input bounded-output (BIBO) stable if for any bounded applied input; the response of the system is also bounded. The knowledge of the impulse response of the systems allows us to specify a new criterion about the stability of a system.

An LTI system is BIBO stable if its impulse response is absolutely integrable on 

The mathematical expression (equation 8.4) is



**Example:**

A system is described by the impulse response. Tell is this system is BIBO stable and verify your conclusion.

|  |  |  |
| --- | --- | --- |
| Commands | Results | Comments |
| syms t  h=exp(-t.^2);  int(abs(h),t,-inf,inf) | ans=pi^(1/2) | Condition 8.4 is fulfilled; hence, the system under consideration is BIBO stable. |

A system is BIBO stable if the condition given in 8..1 is satisfied; that is, we have to examine if its impulse response is absolutely integrable.

In order to verify that this is a BIBO stable system, the bounded input signal is applied to the system. The response of the system is expected to be also bounded.

|  |  |  |
| --- | --- | --- |
| Commands | Results | Comments |
| t=-10:0.1:10;  x1=ones(size(t));  h=exp(-t.^2);  y1=conv(x1,h)\*0.1;  plot(-20:0.1:20,y1,'linewidth',2), grid on;  title('System Response to u(t+10)-u(t-10)' | lab8.bmp | The system response  is computed from the convolution between  and. |

Indeed, the response of the systems is bounded; thus the BIBO stability of the system is verified.

**8.4 Stability Criterion for Discrete Time Systems:**

In the previous section, we have established a criterion about the stability of continuous time LTI systems. More specifically, it was stated that a system is stable if the impulse response of the system is absolutely integrable. For discrete time systems a similar criterion can be established. More specifically, a discrete time linear shift invariant system is stable if and only if its impulse response  is absolutely summable. The mathematical expression (equation 8.2) is



**Example:**

A system is described by impulse response. Tell if this is a BIBO stable system.

A discrete time system is BIBO stable if the condition given in equation 8.2 is satisfied; that is; we examine if the impulse response of the system is absolutely summable.

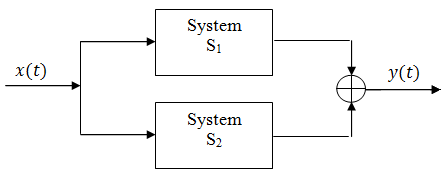
|  |  |  |
| --- | --- | --- |
| Commands | Results | Comments |
| syms n  h=1/(2^n)  symsum(abs(h),n,0,inf) | **ans=2** | Condition 8.4 is not fulfilled; hence, the system is not BIBO stable. |

**In-Lab Tasks**

**Task 01: A system is described by the impulse response. Tell if this is a BIBO stable system.**

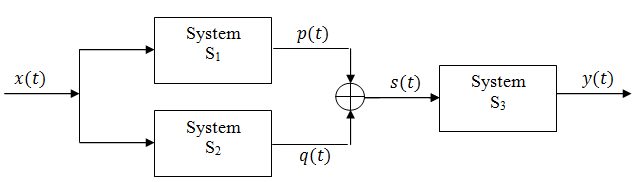
|  |
| --- |
| syms t  h=t^2;  int(abs(h),t,0,inf)  ans= inf  As the answer in infinite, hence the system h(t) is not BIBO stable |

**Task 02:** Suppose that the impulse response of the subsystems S1 and S2 that are connected as shown in figure below are and . Determine if the overall system is BIBO stable.



|  |
| --- |
| step=0.01;  t=0:step:3;    u=ones(size(t));  subplot(3,1,1);  h1=u.\* exp(-3.\*t);  plot(t,h1,'r-.','linewidth',2),grid on;  legend('h1(t)');  xlabel('t')    subplot(3,1,2)  h2=u.\*(t.\*exp(-2.\*t));  plot(t,h2,'b--','linewidth',2),grid on;  legend('h2(t)');  xlabel('t')    h=h1+h2;  subplot(3,1,3)  plot(t,h,'m:','linewidth',2),grid on  legend('h(t)');  xlabel('t')  syms t  f=exp(-3.\*t)+(t.\*exp(-2.\*t));  int(abs(f),t,0,inf)    ans =    7/12  System is BIBO Stable.  Chart  Description automatically generated |

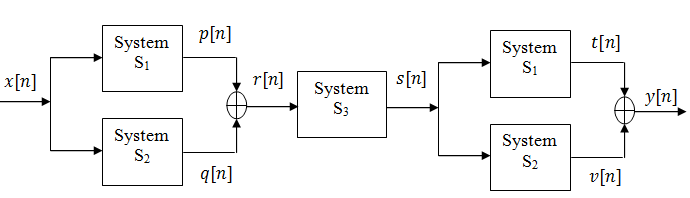
**Task 03:** Suppose that the impulse responses of the sub-systems S1, S2 and S3 that are connected as shown in the figure below are 4 ; 4; and . Compute and plot in the appropriate time interval the impulse response of the overall system and the response of the overall system to the input signal

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1. Make only one file for this task.
2. Call functions within this m-file which is needed.
3. Compute impulse response of the overall system.
4. Compute the response of the overall system to the given input signal
5. Plot all graphs
6. Determine if the overall system is BIBO stable or not.

|  |
| --- |
| close all  clc  clear all  step=0.01;    t1=0:0.01:4;  t2=t1;  t3=t2;  figure();  h1= t1.\*cos(2.\*pi.\*t1);  subplot(3,1,1);  plot(t1,h1,'r-.','linewidth',2),grid on  legend('p(t)');    h2= t2.\*exp(-2.\*t2);  subplot(3,1,2);  plot(t2,h2,'b--','linewidth',2),grid on  legend('q(t)');  title('+','fontsize',16)    h3=ones(size(t3));    h12=h1+h2;  subplot(3,1,3);  plot(t2,h12,'m:','linewidth',2),grid on  legend('s(t)');  title('=','fontsize',16)  %overall impulse response  figure();  th=0:step:8;  h= conv(h12,h3)\*step;  subplot(3,1,1)  plot(th,h,'r-.','linewidth',2),grid on  legend('h(t)')  title('overall impulse response','fontsize',14)  %input signal  tx=0:step:2;  x=tx.\*exp(-2.\*tx);  subplot(3,1,2);  plot(tx,x,'b--','linewidth',2),grid on  legend('x(t)')  title('Input signal','fontsize',14)  %the response of the overall system to the given input signal \_\_\_\_  ty=0:0.01:10;  y=conv(x,h)\*step;  subplot(3,1,3);  plot(ty,y,'m:','linewidth',2),grid on  legend('y(t)')  title('The response of the overall system to the given input signal','fontsize',14)  BIBO STABLE  Chart, line chart  Description automatically generated  Graphical user interface, chart  Description automatically generated |

**Task 04:** Suppose that the impulse responses of the sub-systems S1, S2 and S3 that are connected as shown in the figure below are 2; 2; and 2, respectively. Compute

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1. Make only one file for this task.
2. Call functions within this m-file which is needed.
3. Compute impulse response of the overall system.
4. Compute the response of the overall system to the given input signal
5. Plot all graphs
6. Determine if the overall system is BIBO stable or not.

|  |
| --- |
| n = 0:2;  h1 = [2,3,4];  h2 = [-1,3,1];  h3 = [1,1,-1];    subplot(5,1,1)  stem(n,h1,'fill','linewidth',2),grid on  legend('p[n]')    subplot(5,1,2)  stem(n,h2,'fill','linewidth',2),grid on  legend('q[n]')    h12=h1+h2;  subplot(5,1,3)  stem(n,h12,'fill','linewidth',2),grid on  legend('r[n]')  subplot(5,1,4)  stem(n,h3,'fill','linewidth',2),grid on  legend('h3[n]')    n123 = 0:4;  h123 = conv (h12,h3);  subplot(5,1,5)  stem(n123,h123,'fill','linewidth',2),grid on  legend('s[n]')    figure();  nt=0:6;  h1231=conv(h123,h1);  subplot(5,1,1)  stem(nt,h1231,'fill','linewidth',2),grid on  legend('t[n]')    nv=0:6;  h1232=conv(h123,h2);  subplot(5,1,2)  stem(nv,h1232,'fill','linewidth',2),grid on  legend('v[n]')      nh=0:6;  h=h1231+h1232;  subplot(5,1,3)  stem(nh,h,'g--','fill','linewidth',2),grid on  legend('h[n]')    xn=0:1;  x=ones(size(xn));  subplot(5,1,4)  stem(xn,x,'r:','fill','linewidth',2),grid on  legend('x[t]');    ty= 0:7;  y= conv(x,h);  subplot(5,1,5)  stem(ty,y,'r:','fill','linewidth',2),grid on  legend('y[n]')    syms n  ans=symsum (abs(h),n,0,inf)    ans =    [ Inf, Inf, Inf, Inf, Inf, Inf, Inf]  Calendar  Description automatically generated with low confidence  Chart, diagram  Description automatically generated with medium confidence |

**Post-Lab Tasks**

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| In this lab, we implemented various properties of convolution which includes commutative, associative, distributive and identity property. We also implemented them on different types of system: cascaded, parallel, mixed and feedback system and obtained the output using those properties in these systems. |

**Critical Analysis / Conclusion**

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| **Lab Assessment** | | |
| **Pre-Lab** | **/1** | **/10** |
| **In-Lab** | **/5** |
| **Critical Analysis** | **/4** |
| **Instructor Signature and Comments** | | |